

KEY REMINDERS

- An **expression** has no equals sign (e.g. $3x + 2$). An **equation** does (e.g. $3x + 2 = 11$).
- To **expand** a bracket, multiply everything inside by what's outside: $a(b + c) = ab + ac$.
- To **solve** an equation, do the same operation to both sides until x is on its own.
- If x appears on both sides, move all the x -terms to one side first.

1 • Writing Expressions and Equations

Objective: translate a worded statement into algebraic form.

WORKED EXAMPLE

Statement

"A taxi charges a \$4 fixed fee plus \$2 for every kilometre travelled."

Let

k = number of kilometres

Expression

$\text{cost} = 4 + 2k$

1. Write an expression for "a number, n , increased by 8."
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2. Write an expression for "three times a number, n , decreased by 5."
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3. A market stall sells apples for \$3 each. Write an expression for the cost of buying a apples plus a \$1 bag fee.
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2 • Substitution

Objective: substitute numbers into an algebraic expression and evaluate it.

WORKED EXAMPLE

Given

$a = 3, b = -2$. Evaluate $2a + b$

Substitute

$2(3) + (-2)$

Simplify

$6 - 2 = 4$

- Use $a = 3, b = -2, c = 5$ for questions 4–6.
4. Evaluate $2a + b$
-
5. Evaluate $b^2 + c$
-
6. Evaluate $3a - 2b$
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3 · Expanding Brackets

Objective: expand a single bracket and expand two brackets.

WORKED EXAMPLE — single bracket
Expand $4(x + 3)$
Multiply each term $4 \times x = 4x, \quad 4 \times 3 = 12$
Answer **$4x + 12$**

WORKED EXAMPLE — two brackets
Expand $(x + 2)(x + 5)$
Multiply every term in the first bracket by every term in the second
 $(x)(x) + (x)(5) + (2)(x) + (2)(5) = x^2 + 5x + 2x + 10$
Collect like terms **$x^2 + 7x + 10$**

7. Expand: $3(x + 4)$

8. Expand: $-2(x - 5)$

9. Expand: $(x + 3)(x + 2)$

10. Expand: $(x - 4)(x + 6)$

4 · Solving Equations with One Unknown

Objective: solve a linear equation where x appears once.

WORKED EXAMPLE
Solve $3x + 5 = 20$
Subtract 5 $3x = 15$
Divide by 3 **$x = 5$**

11. Solve: $3x + 5 = 20$

12. Solve: $2x - 7 = 11$

13. Solve: $5(x - 2) = 15$

5 • Equations with the Unknown on Both Sides

Objective: solve a linear equation with x-terms on both sides.

WORKED EXAMPLE

Solve $5x + 3 = 2x + 18$
Subtract $2x$ from both sides $3x + 3 = 18$
Subtract 3 $3x = 15$
Divide by 3 $x = 5$

14. Solve: $5x + 3 = 2x + 18$

15. Solve: $7x - 4 = 3x + 12$

16. Solve: $4(x + 1) = 2x + 10$

6 • Algebraic Fractions

Objective: simplify algebraic fractions and solve equations that contain them.

WORKED EXAMPLE

Simplify $\frac{8x^2}{4x}$
Cancel common factor $4x$ $2x$

17. Simplify: $\frac{6x}{9}$

18. Simplify: $\frac{x^2 - 9}{x + 3}$ (hint: factorise the top first)

19. Write as a single fraction: $\frac{x}{3} + \frac{x}{4}$

20. Solve: $\frac{x}{2} + \frac{x}{3} = 5$

Challenge — Bringing It Together

Objective: combine two or more of today's skills in one problem.

CHALLENGE 1

A rectangle has a width of x cm and a length 4 cm longer than its width. Its perimeter is 28 cm.

(a) Write an equation for the perimeter in terms of x .

(b) Solve your equation to find x .

CHALLENGE 2

Solve: $3(x - 1) = x + 2(x + 4) - 15$

Expand both sides fully before you start collecting terms.

SELF-CHECK

Before you hand this in, circle how confident you feel with each topic today:

Writing expressions: (Not yet / Getting there / Confident)

Substitution: (Not yet / Getting there / Confident)

Expanding brackets: (Not yet / Getting there / Confident)

Solving equations: (Not yet / Getting there / Confident)

Equations, both sides: (Not yet / Getting there / Confident)

Algebraic fractions: (Not yet / Getting there / Confident)

1 • Writing Expressions and Equations

1. $n + 8$
2. $3n - 5$
3. $3a + 1$

2 • Substitution (a = 3, b = -2, c = 5)

4. $2(3) + (-2) = 4$
5. $(-2)^2 + 5 = 4 + 5 = 9$
6. $3(3) - 2(-2) = 9 + 4 = 13$

3 • Expanding Brackets

7. $3x + 12$
8. $-2x + 10$
9. $x^2 + 5x + 6$
10. $x^2 + 2x - 24$

4 • Solving Equations, One Unknown

11. $3x = 15 \rightarrow x = 5$
12. $2x = 18 \rightarrow x = 9$
13. $x - 2 = 3 \rightarrow x = 5$

5 • Equations, Unknown on Both Sides

14. $3x = 15 \rightarrow x = 5$
15. $4x = 16 \rightarrow x = 4$
16. $4x + 4 = 2x + 10 \rightarrow 2x = 6 \rightarrow x = 3$

6 • Algebraic Fractions

17. $2x/3$
18. $(x^2-9)/(x+3) = (x+3)(x-3)/(x+3) = x - 3$
19. $4x/12 + 3x/12 = 7x/12$
20. Common denominator 6: $3x + 2x = 30 \rightarrow 5x = 30 \rightarrow x = 6$

Challenge

1. (a) $2(x) + 2(x + 4) = 28$ (b) $4x + 8 = 28 \rightarrow 4x = 20 \rightarrow x = 5$ (width 5 cm, length 9 cm)
2. $3x - 3 = x + 2x + 8 - 15 \rightarrow 3x - 3 = 3x - 7 \rightarrow -3 = -7$, which is never true $\rightarrow$ **no solution** (the x-terms cancel — a good discussion point on why)